True, Equivalent, and Calibrated Airspeeds

The energy equation for adiabatic flow of an ideal gas is

$$\frac{v^2}{2} + h = \text{constant}$$

where v is the flow velocity and h is the specific enthalpy. This implies that if a flow with free stream velocity v_0 and enthalpy h_0 is brought to rest adiabatically, and if h_1 is the enthalpy of the resting gas, then

$$h_1 - h_0 = \frac{v_0^2}{2}$$

Also, for an ideal gas we have $dh/dT = c_p$, where the specific heat c_p is essentially constant for relatively small changes in temperature, so this equation can be written as

$$T_1 - T_0 = \frac{v_0^2}{2c_p}$$

Now, the speed of sound in an ideal gas is given by $a = \sqrt{\gamma R T}$ where g = c_p/c_v is the ratio of specific heats and R is the gas constant, so we can express any given temperature T in terms of the speed of sound at that temperature. With these substitutions, the preceding equation becomes

$$a_1^2 - a_0^2 = \gamma R \frac{v_0^2}{2c_p}$$

Also, for an ideal we have $R = c_p - c_v$, so we can make this substitution, divide through by a_0^2, and re-arrange terms to give

$$\frac{a_1^2}{a_0^2} = 1 + \left(\frac{\gamma - 1}{2}\right)\frac{v_0^2}{a_0^2}$$

The ratio v_0/a_0 is, by definition, the Mach number M of the free stream, and since temperature is proportional to the square of the speed of sound, the ratio $(a_1/a_0)^2$ equals the ratio of temperatures T_1/T_0. Hence we have

$$\frac{T_1}{T_0} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

Furthermore, using the ideal gas equation p = rRT we can substitute for the temperatures to give

$$\frac{p_1/\rho_1}{p_0/\rho_0} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

As discussed in The Speed of Sound, for an adiabatic quasi-static (i.e., isentropic) process, the ratio p/rg is constant. Thus if we re-write the above equation as

$$\left(\frac{p_1/\rho_1{}^{\gamma}}{p_0/\rho_0{}^{\gamma}}\right)^{1/\gamma}\left(\frac{p_1}{p_0}\right)^{1-1/\gamma} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

the first factor on the left side is unity, and we arrive at the formula for the isentropic pressure rise of an ideal fluid being brought to a stop from a Mach number M

$$\left(\frac{p_1}{p_0}\right)^{(\gamma-1)/\gamma} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

Solving this equation for M and then multiplying through by the freestream speed of sound $a_0 = \sqrt{\gamma R T_0}$, we have the formula for the true airspeed $v_{true} = v_0$ as a function of the freestream static pressure p_0, the total (stagnation) pressure p_1, and the freestream static temperature T_0

$$v_{true} = \sqrt{\left(\frac{2\gamma R T_0}{\gamma-1}\right)\left[\left(\frac{p_1}{p_0}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

If the pressure ratio p_1/p_0 is known but the freestream static temperature T_0 is not (as is sometimes the case with primitive instrumentation), we can agree by convention to simply use for T_0 the standard sea level atmospheric temperature

$$T_{standard} = 518.67\ R$$

When this is done, the result is called equivalent airspeed. The factor of $\sqrt{\gamma R T_0}$ in the above equation is then simply taken to be the speed of sound at standard sea level temperature, so the equation for equivalent airspeed is

$$v_{equivalent} = a_{standard}\sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_1}{p_0}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

where

$$a_{standard} = 1116 \text{ ft/sec} = 661.47 \text{ knots}$$

Of course, the square root in this expression is simply the true Mach number, so equivalent airspeed can also be written as

$$v_{equivalent} = a_{standard}M = a_0\sqrt{\frac{T_{standard}}{T_0}}\,M$$

Thus, noting that a_0M is the true airspeed, and letting q denote the ratio $T_0/T_{standard}$, we have

$$v_{equivalent} = \frac{v_{true}}{\sqrt{\theta}}$$

Sometimes we lack not only the static temperature, but the pressure ratio as well. With certain kinds of primitive instrumentation systems we can measure only the difference $p_1 - p_0$ between the total and static pressures. (This difference is sometimes called the impact pressure, which is the same as the dynamic pressure $1rv^2$ for incompressible flow, but *not* for compressible flow.) Even if we can re-write the ratio p_1/p_0 as $(p_1-p_0)/p_0 + 1$, we would still need to know the static pressure p_0 (in addition to the impact pressure) in order to compute equivalent airspeed. However, lacking the static pressure, we can agree by convention to simply use the standard sea level static pressure in place of p_0. When this is done, we get the following formula for calibrated airspeed as a function only of the impact pressure

$$v_{calibrated} = a_{standard}\sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_1-p_0}{p_{standard}}+1\right)^{(\gamma-1)/\gamma} - 1\right]}$$

where

$$p_{standard} = 14.696 \text{ psia}$$

Given the definition of calibrated airspeed, we sometimes need to compute it based on the actual measured values of the static pressure p_0 and freestream Mach number M. (Of course, knowing p_0 and M, we could compute equivalent airspeed, but convention may still force us to deal with calibrated airspeed.) For convenience, we will set g = 1.4, which is the value for atmospheric air. Then p_1 is given in terms of p_0 and Mach by the relation

$$p_1 = p_0\left(1+\frac{M^2}{5}\right)^{7/2}$$

Substituting this into the calibrated airspeed equation gives

$$v_{\text{calibrated}} = a_{\text{standard}}\sqrt{5}\sqrt{\left(\delta\left[\left(1+\frac{M^2}{5}\right)^{7/2}-1\right]+1\right)^{2/7}-1}$$

where d denotes the pressure ratio p_0/p_{standard}. For values of M less than $\sqrt{5}$ this can be expanded into a convergent power series in M, the first few terms of which are

$$v_{\text{calibrated}} = a_{\text{standard}}\sqrt{\delta}\,M\left[1 + \frac{1}{8}(1-\delta)M^2 + \frac{3}{640}\left(1-10\delta+9\delta^2\right)M^4 + \dots\right]$$

In terms of the true airspeed this is

$$v_{\text{calibrated}} = v_{\text{true}}\sqrt{\frac{\delta}{\theta}}\left[1 + \frac{1}{8}(1-\delta)M^2 + \frac{3}{640}\left(1-10\delta+9\delta^2\right)M^4 + \dots\right]$$

For small Mach numbers (much less than 1) the zeroth order term of the expansion is often accurate enough for practical purposes, but for Mach numbers approaching 1 it is necessary to take account of the higher-order terms - or else simply use the exact analytical expression.

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